

1N-83

45755

P-14

Reliability and Cost: A Sensitivity Analysis

(NASA-TN-105293) RELIABILITY AND COST: A
SENSITIVITY ANALYSIS (NASA) 14 D CSU OSC

101-33014

03/83 0045755
Unclass

Ronald C. Suich
California State University
Fullerton, California

and

Richard L. Patterson
Lewis Research Center
Cleveland, Ohio

October 1991

NASA

Reliability and Cost: A Sensitivity Analysis

Ronald C. Sulch
California State University
Fullerton, CA 92634

Richard L. Patterson
NASA Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

In the design phase of a system, how does a design engineer or manager choose between a subsystem with .990 reliability and a more costly subsystem with .995 reliability? When is the increased cost justified?

High reliability is not necessarily an end in itself but may be desirable in order to reduce the expected cost due to subsystem failure. However, this may not be the wisest use of funds since the expected cost due to subsystem failure is not the only cost involved. The subsystem itself may be very costly. We should not consider either the cost of the subsystem or the expected cost due to subsystem failure separately but should minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure.

ASSUMPTIONS AND NOTATION

In this paper assume perfect switching devices (if needed) of negligible cost and independence of the subsystem modules.

NOTATION

n	number of modules in the subsystem
k	minimum number of good modules for the subsystem to be good
r	reliability of the whole system for other than failure of the subsystem
r_s	reliability of the subsystem
c_1	loss due to failure of the subsystem
c_2	loss due to subsystem output at v_c (for models 3, 4, and 5)
c_3	cost of a one module subsystem capable of full output
c_k	cost of a module in a k-out-of-n:G subsystem when k is fixed (see later discussion)
$g(k)$	function which relates cost of subsystem to the number of modules in the subsystem
v_c	fraction of subsystem output necessary so that the mission is not a failure
p	probability that a module is good
q	probability that a module fails or $1-p$
C	the total of the cost of the subsystem itself plus the expected loss due to subsystem failure
λ	failure rate of a module (models 4 and 5)
T_0	mission time

INTRODUCTION

Since expected value is an important ingredient in our quest for finding the best subsystem, consider the expected cost due to subsystem failure denoted as $E\{\text{cost due to subsystem failure}\}$. As with all expected values, it depends upon both the dollar cost and the probability of its occurrence. If we let Pr mean "probability of", then $E\{\text{cost}\} = \text{cost} \times Pr\{\text{cost occurrence}\}$. Let c_1 be the cost due to failure of the subsystem, including all costs incurred by subsystem failure (but not the cost of the subsystem itself). This number could be the entire cost of the main system (or even greater) if failure of the subsystem resulted in failure of the main system. In other instances c_1 would be less than the cost of the main system, e.g., failure of the subsystem resulted in only a partial failure of the main system.

Now the expected cost due to subsystem failure is c_1 times the probability that this cost will be experienced. To experience a cost due to subsystem failure, two events must occur, namely: 1. the main system must be good, and 2. the subsystem must fail. For example, if the main system (a rocket) is not good (e.g. a launch is canceled or the rocket explodes for some reason other than for the subsystem being considered), then a cost due to subsystem failure cannot occur. So for our expected cost we want to consider the $Pr\{\text{main system good and subsystem failure}\}$. Let r be the reliability of the main system (for other than failure of the subsystem) and let r_s be the reliability of the subsystem. We will also use the fact that $Pr\{A \text{ and } B\} = Pr\{A \cap B\} = Pr\{A\}Pr\{B | A\}$.] Then

$$E\{\text{cost due to subsystem failure}\} = c_1 Pr\{\text{main system good} \cap \text{subsystem failure}\} = c_1 Pr\{\text{main system good}\} Pr\{\text{subsystem failure} | \text{main system good}\} = c_1 r(1-r_s) = rc_1(1-r_s).$$

We can minimize this expected cost by building a subsystem with an extremely low probability of failure (high reliability). However, it is not clear that we should build the most reliable subsystem possible since this will minimize only the expected cost due to subsystem failure but does not consider the cost of building the subsystem itself. We should not consider the two costs separately. We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure. The total cost to be minimized is

$$\begin{aligned} C &= \text{cost of the subsystem} + E\{\text{cost due to subsystem failure}\} \\ &= \text{cost of the subsystem} + rc_1(1-r_s) \end{aligned} \quad (1).$$

In minimizing cost C we see that we are balancing the cost of the subsystem and the expected cost due to subsystem failure.

SELECTING THE BETTER SUBSYSTEM

Suppose that we are considering two subsystems. Subsystem 1, which costs \$200,000, has a .97 reliability. Subsystem 2, with a cost of \$100,000, has a .94 reliability. Without further analysis, there is no clear "best" subsystem and the choice is often based upon the amount budgeted for the subsystem.

Assume that the two subsystems under consideration will be part of a main system which has a reliability (exclusive of the subsystem under consideration) of $r = .96$. We'll further assume that failure of the subsystem will result in a cost of $c_1 = \$10,000,000$. Let us first compare the $E\{\text{cost due to subsystem failure}\}$ for each of the two subsystems.

$$\begin{aligned} \text{For subsystem 1,} \\ E\{\text{cost due to subsystem failure}\} &= rc_1 Pr\{\text{subsystem failure}\} \\ &= rc_1(1-r_{s1}) \\ &= .96 \times \$10,000,000 \times .03 = \$288,000. \end{aligned}$$

For subsystem 2,

$$\begin{aligned} E\{\text{cost due to subsystem failure}\} &= r c_1 (1-r_{s2}) \\ &= .96 \times \$10,000,000 \times .06 = \$576,000. \end{aligned}$$

Subsystem 2 has a higher expected cost than subsystem 1. However, since 2 is also less expensive, we need to compare the overall expected cost, C , for 1 and for 2.

For subsystem 1,

$$C_{s1} = \$200,000 + \$288,000 = \$488,000.$$

For subsystem 2,

$$C_{s2} = \$100,000 + \$576,000 = \$676,000.$$

Since $C_{s1} < C_{s2}$, we select subsystem 1 over subsystem 2.

For further information on expected values or on selecting the best subsystem, see [3].

K-OUT-OF-N:G SUBSYSTEMS

In this article we'll direct our attention to a specific type of subsystem, called a k-out-of-n:G subsystem. Such a subsystem has n modules, of which k are required to be good for the subsystem to be good. As an example consider the situation where the engineer has a certain power requirement. He may meet this requirement by having one large power module, two smaller modules, etc. The number of modules required is called k . For example, the engineer may decide that $k = 4$. Then each module is $1/4$ of the full required power. Therefore, the subsystem must have 4 or more modules for the full required power. The number of modules used in the subsystem is called n . For example, an $n = 6$ and $k = 4$ subsystem would have 6 modules each of $1/4$ power and thus would have the output capability of 1.5 times the required power. The engineer chooses n and k . Selection of the different values of n and k results in different subsystems, each with different costs and reliabilities. Since each n and k yields different subsystems with different costs, we can choose the subsystem (the n and k) which will minimize cost C .

MODEL 1

The simplest k-out-of-n: G model is one where the modules are independent and all have common probability of being good p and common probability of failure $q = 1-p$. Let X count the number of good modules. Now

$$\begin{aligned} E\{\text{cost due to subsystem failure}\} &= r c_1 \Pr\{\text{subsystem failure}\} \\ &= r c_1 \Pr\{X < k\} = r c_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}. \end{aligned} \quad (2)$$

Recall that $C = \text{cost of subsystem} + E\{\text{cost due to subsystem failure}\}$. We therefore need to also consider the cost of the subsystem. First consider a simple situation where k is fixed. Here we are free to choose n . Then $n-k$ will be the redundancy or number of spares in the subsystem. If each module costs c_1 , then the cost of subsystem = $n c_1$. Using this with (2) we obtain

$C = \text{cost of subsystem} + E\{\text{cost due to subsystem failure}\}$

$$= n c_s + r c_f \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}.$$

We wish to find the n which minimizes cost C .

The authors have written a BASIC program (QuickBASIC 4.5) to find the n which minimizes C . Additionally this program will, if you desire, graph C as a function of either p or c_f . The program will plot the best subsystems (i.e. the ones with the lowest C 's) over ranges of p or c_f . This allows you to not only select the best subsystem for a particular value of p or c_f but also to view what happens to C for nearby values of p or c_f .

As an example, consider the situation when $k = 1$, where only one module is required to be operational for the subsystem to be operational. The reliability of this single module is estimated to be .95 ($p = .95$). Let the reliability of the system for other than failure of the subsystem be .9, ($r = .9$). The cost of one module is 1 ($c_s = 1$) million dollars (throughout the remainder of the paper all costs will be in millions of dollars). The cost due to failure of this subsystem is 10 ($c_f = 10$).

Figure 1 shows a plot of C for p ranging from .79 to .99 and n 's of 1 through 4. When the reliability of a single module $p = .95$, $n = 1$ has the lowest value of C . Therefore the best subsystem in this case is one with no spares. We see from figure 1 that the $n = 1$ subsystem (no spares) has the lowest value of C for any $p > .87$. If $p < .87$, then $n = 2$ (one spare) has the lowest value of C . For $p < .79$, we would view the graph over the range of $p < .79$.

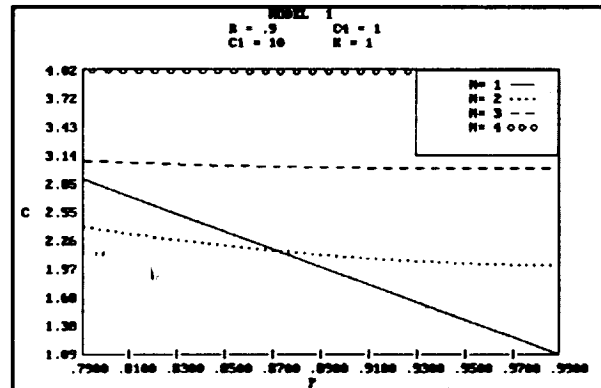


Figure 1

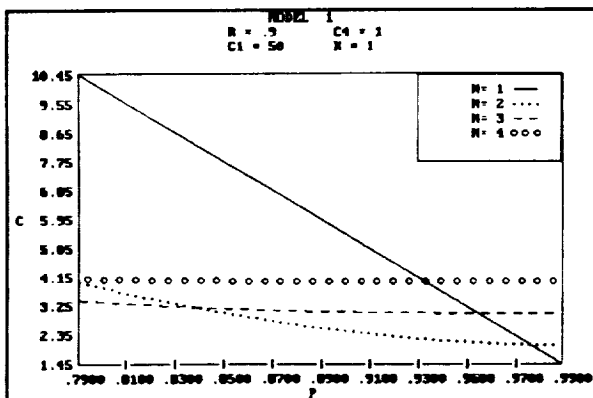


Figure 2

Now suppose instead that c_f (cost due to failure of the subsystem) is 50. Figure 2 shows the plot of C for $c_f = 50$. We first note that if $p = .95$, then the $n = 2$ subsystem is the best. Comparing figures 1 and 2 (at $p = .95$) we see that the larger value of c_f (in figure 2) requires a larger value of n . This principle holds in general. If the cost of subsystem failure increases then more redundancy is required. If $.83 < p < .98$, figure 2 shows that the $n = 2$ subsystem is best. If p is below .83 then more redundancy ($n=3$) is required. If $p > .98$, then no redundancy ($n=1$) is required.

MODEL 2

If, in model 1, we are also free to choose k in our subsystem, then we have model 2. Let c_3 be the cost of a subsystem consisting of exactly one module. Further suppose that the cost of a subsystem with exactly k modules is $c_3 g(k)$. Here $g(k)$ is the factor which measures the (generally) increased cost of building a subsystem consisting of k smaller modules rather than one large module. If $g(k) = 1$ for all k , then a subsystem of k modules costs the same as a subsystem consisting of a single module. Any $g(k)$ may be used. For example, if a subsystem of 2 smaller modules costs 4 times as much as a single module subsystem then $g(2) = 4$. Therefore this subsystem would cost $c_3 g(k) = c_3 g(2) = 4c_3$. If a subsystem of 3 smaller modules costs 7 times as much as a single module subsystem then $g(3) = 7$. Other values for $g(k)$ may be defined in a similar manner. Therefore, in the above example, $g(1) = 1$, $g(2) = 4$, $g(3) = 7$, etc. We also assume that each module in the subsystem costs $c_3 g(k)/k$, which is $1/k$ of the total cost for k modules. Since we have a total of n modules in the subsystem, then the cost of the subsystem = $nc_3 g(k)/k$. Using this with (2) we obtain

$$C = \text{cost of subsystem} + E\{\text{loss due to subsystem failure}\}$$

$$= n c_3 g(k)/k + r c_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}.$$

For any particular situation with given values of c_1 , c_3 , r , p and $g(k)$ we use the BASIC program to select the n and k to minimize C as given above. There are two options for $g(k)$ built into the BASIC program. You may choose either $g(k) = (1+b)g(k-1)$ or $g(k) = k(1/k)^c$, where you are free to set b or c .

If you believe that the cost of building a subsystem of k modules increases (or decreases) linearly with k , then you would choose the first option $g(k) = (1+b)g(k-1)$, with $b > 0$ ($b < 0$). For example, if building a subsystem of two smaller modules costs 20% more than building a single module subsystem, 3 modules costs 20% more than a subsystem of two modules, etc., then let $b = .2$. If you believe that the cost of building a subsystem is exponentially proportional to the number of modules in the subsystem then you would choose the second option $g(k) = k(1/k)^c$. For example, consider building a space electrical power subsystem. A rough rule of thumb says that the cost of smaller modules for a space electrical power subsystem is proportional to the electrical power raised to the .7, i.e., $g(k) = k(1/k)^{.7}$. Therefore, a subsystem consisting of a single module capable of full power costs $c_3 g(1) = c_3 1(1/1)^{.7} = 1.0c_3$. A subsystem consisting of 2 modules, each of $1/2$ power, costs $c_3 g(2) = c_3 2(1/2)^{.7} = 1.23c_3$ to build, etc. An $n = 3$ and $k = 2$ subsystem, (one having 3 modules each of $1/2$ power) costs $nc_3 g(k)/k = 3c_3 (1/2)^{.7} / 2 = 3c_3 \times 1.23 / 2 = 1.85c_3$ to build.

As an example of model 2, suppose we are building a space electrical power subsystem. The cost due to subsystem failure, c_1 , is 240. Let the reliability of the system for other than failure of the subsystem be .9 ($r = .9$). Suppose that the cost of building a single module capable of full power is 1 ($c_3 = 1$). Using the rule of thumb stated above, we use the option for $g(k)$ with $c = .7$. All of the above values are entered into the BASIC program as parameters. An estimate of p , the reliability of an individual module, is .96. If we are unsure of this estimate, we can use the BASIC program to view (figure 3) the best subsystems over p ranging from .89 to .99.

From figure 3, at $p = .96$, the $n = 2, k = 1$ subsystem is best (lowest value of C). If $p < .95$, the $n = 4, k = 2$ subsystem is best. Note this is a flatter curve over the range of p , indicating a low value for C over a wide range of p .

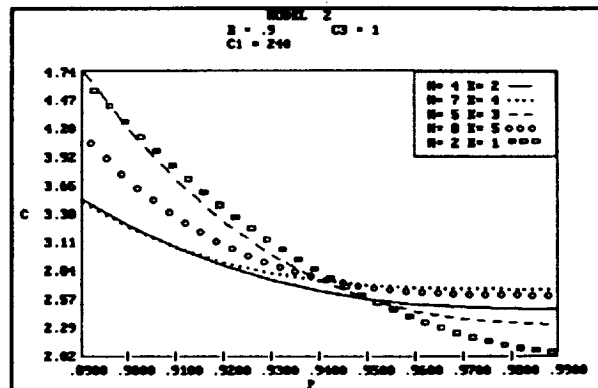


Figure 3

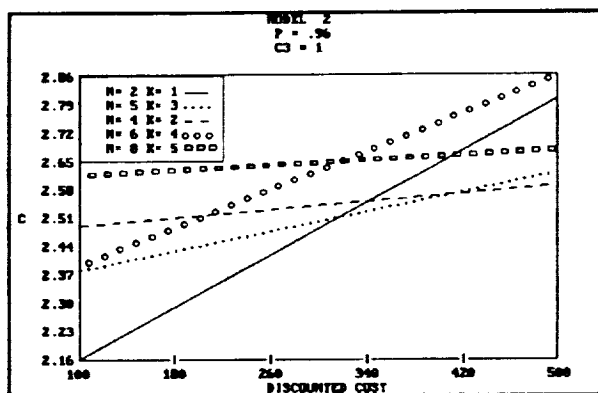


Figure 4

For the same example, suppose we wish to view what happens to C as c_1 varies. Figure 4 (from the BASIC program) shows, if c_1 is below 310, then the $n = 2, k = 1$ subsystem is best. However, for $310 < c_1 < 400$, the $n = 5, k = 3$ subsystem is the best. For $c_1 > 400$ the $n = 4, k = 2$ subsystem is the best. This type of analysis could be used whenever you are unsure of c_1 and wish to consider results over a range of values.

MODEL 3

Figure 5 shows the loss due to subsystem failure, where v is the ratio of the actual output of the subsystem to the specification output. If v drops below some critical value v_c , the mission is a complete failure and the loss is c_1 . However, if v is at v_c , then the loss is only c_2 . As v increases above v_c , this loss decreases until there is no loss at full output.

Although h is linear in figure 5 other loss functions, e.g., a decreasing multi-step function, are appropriate. If $h(v) = a - av$, $v_c < v < 1$, $a = c_2/(1-v_c)$, (1) becomes

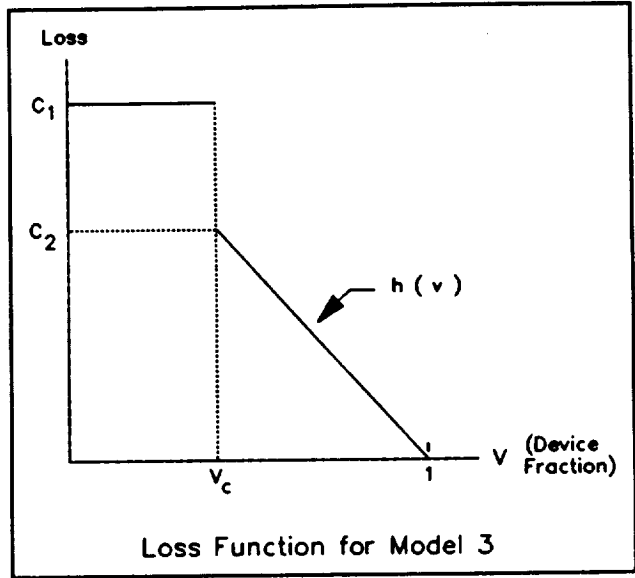


Figure 5

$$C = nc_3 g(k)/k+r \quad c_1 \sum_{x=0}^{x \leq kv_c} \binom{n}{x} p^x q^{n-x} + r \sum_{x \geq kv_c}^{k-1} \binom{n}{x} p^x q^{n-x} (a - ax/k).$$

The third term on the rhs is expected loss due to partial failure of the subsystem. Again we can find, by means of the BASIC program, the n and k which minimize C .

MODEL 4

Suppose in model 3 (with $c_1 = c_2$) that mission time is also important. If modules fail exponentially with failure rate λ , then the probability of a module still operating successfully at time t is $\exp(-\lambda t)$. Let $f(x, t)$ be the joint probability density function of x successes and time t . We will use the fact that $f(x, t) = g(x) f(t | x)$. Now $f(t | x)$ is the time at which the x^{th} success occurs (the waiting time for the $(n-x)^{\text{th}}$ failure), given that $n-x$ failures have occurred before mission time T_0 . Then

$$f(t | x) = L(t | x) / \int_0^{T_0} L(t | x) dt \quad 0 < t < T_0$$

$$\text{where } L(t | x) = \frac{n!}{x!(n-x-1)!} [\exp(-\lambda t)]^x \lambda \exp(-\lambda t) [1 - \exp(-\lambda t)]^{n-x-1}.$$

$$\therefore f(x, t) = f(t | x) g(x)$$

$$\text{where } g(x) = \binom{n}{x} [\exp(-\lambda T_0)]^x [1 - \exp(-\lambda T_0)]^{n-x} \quad x=0, 1, \dots, n.$$

Note: $g(x)$ is the probability of exactly x successes in n modules at mission time T_0 .

$L(t|x)$ is a probability density function (pdf) which, when integrated from t_1 to t_2 , yields the probability that the $(n-x)^{\text{th}}$ failed module (the last module which can fail with x remaining good modules) will occur at a time between t_1 and t_2 . We can write $L(t|x) = R \cdot n \cdot S$ where

$$R = \binom{n-1}{x} [\exp(-\lambda t)]^x [1 - \exp(-\lambda t)]^{n-x-1}$$

$$S = \lambda \exp(-\lambda t).$$

R is a pdf, which when integrated from 0 to t , yields the probability that, with $n-1$ modules, $(n-x-1)$ failed modules and x good modules will occur before time t . S is a pdf, which when integrated from t_1 to t_2 , yields the probability that a module will fail between time t_1 and t_2 . Since any of the n modules can fail

at time t , we multiply R by $n \cdot S$ to obtain $L(t|x)$. Now $\int_0^{T_0} L(t|x) dt$ gives the probability that the $(n-x)^{\text{th}}$ failed module occurs in $(0, T_0)$. Since we wish to define $f(t|x)$ as a probability density function on $0 < t$

$< T_0$, we must have $\int_0^{T_0} f(t|x) dt = 1$, and so we divide $L(t|x)$ by $\int_0^{T_0} L(t|x) dt$ to obtain $f(t|x)$.

If the output fraction is v_c at the start of the mission, our loss is c_2 . As v increases above v_c , then this loss decreases until there is no loss at full output. With output at or above v_c , losses decrease with increasing time until there is no loss beyond mission time T_0 . Additionally, for any given t , $h(v,t)$ decreases as v increases above v_c .

Consider now a general loss function $h(v,t)$ [not necessarily the one illustrated by figure 6]. Again, for a given t , h takes on values only for $v = x/k$.

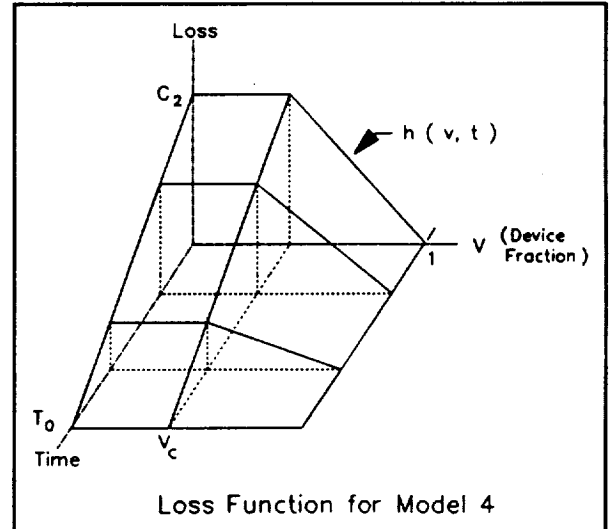


Figure 6

Now (1) becomes

$$C = nc_3 g(k)/k + r \sum_{x=0}^n \int_0^{T_0} h(x/k, t) f(x, t) dt. \quad (3)$$

If we let

$$h(x/k, t) = d(x/k) \sum_{j=0}^m b_j t^j \quad (4)$$

$$A = \sum_{i=0}^{n-x-1} (-1)^{i+1} \binom{n-x-1}{i} [\lambda(x+i+1)^{-1} [\exp(-\lambda T_0(x+i+1)) - 1]]$$

$$B = \exp(-\lambda T_0(x+i+1)) [T_0(\lambda(x+i+1))^{-1} + (\lambda(x+i+1))^{-2}] - (\lambda(x+i+1))^{-2}$$

then, after integrating, (3) becomes

$$C = nc_3 g(k)/k + r \sum_{j=0}^m b_j \left\{ \sum_{x=0}^{k-1} J(x) d(x/k) \sum_{i=0}^{n-x-1} (-1)^{i+1} \binom{n-x-1}{i} \right. \\ \left. [\exp(-\lambda T_0(x+i+1)) \sum_{s=0}^j (j! T_0^{-s}) [(j-s)! (\lambda(x+i+1))^{s+1}]^{-1} \right. \\ \left. - j! [\lambda(x+i+1)]^{-j-1} \right\} \quad \text{where } J(x) = [g(x)]^{-1} A \quad (5)$$

We wish to find the n and k which minimize C . Minimizing C in (5) is appropriate for any loss function, $h(\cdot)$, of the form given in (4). Using the loss function given in figure 6, for $0 \leq x < kv_c$, $d(x/k) = 1$, $m = 1$, $b_0 = c_2$ and $b_1 = -c_2 T_0^{-1}$. For $kv_c \leq x \leq k-1$ we have $d(x/k) = 1 - x/k$, $m = 1$, $b_0 = a$ and $b_1 = -a T_0^{-1}$ where $a = c_2(1-v_c)^{-1}$ with $0 < v_c < 1$. Using (5) we obtain

$$C = nc_3 g(k)/k \\ + r \left\{ c_2 \sum_{x=0}^{x < kv_c} A - c_2 T_0^{-1} \sum_{x=0}^{x < kv_c} J(x) \sum_{i=0}^{n-x-1} (-1)^{i+1} \binom{n-x-1}{i} B \right. \\ \left. + a \sum_{x \geq kv_c}^{k-1} J(x) (1-x/k) A \right. \\ \left. - a T_0^{-1} \sum_{x \geq kv_c}^{k-1} J(x) (1-x/k) \sum_{i=0}^{n-x-1} (-1)^{i+1} \binom{n-x-1}{i} B \right\}$$

MODEL 4 APPLICATIONS

Model 4 might reasonably be applied to non-recoverable systems which, at the end of their service life, have no intrinsic or salvage value or which are prohibitively expensive to recover. Examples include undersea sonar systems anchored in deep water, instrument/telemetry packages located in remote regions or communications satellites in geosynchronous orbit. For a geosynchronous communications satellite a number of subsystems could be chosen as an example. Let us examine the satellite power system which can be divided into smaller identical modules. We again use the rule of thumb which says that the cost of a space power subsystem is proportional to the electrical power raised to the .7 ($g(k) = k(1/k)^{.7}$). Suppose that the mission life is 7 years and the reliability of the satellite (exclusive of the power subsystem) over the mission life is .90. Because the satellite needs power for stationkeeping, computers and cooling, at least 10% of the specification power is needed for the satellite to survive. Therefore, v_c is 0.1. The satellite generates \$2 million per month revenue. In the event of satellite failure, a new satellite could be launched within two years at a cost of \$115 million. Therefore c_1 (or c_2) = 163 (115 plus 48 in lost revenue). Here we will assume that revenue is roughly proportional to power, i.e., if a module of the power subsystem fails, then one or more channels are no longer available. We estimate failure rate, λ , as $3.5(10^{-6})$ failures per hour (hr^{-1}) and again use the BASIC program to view C over a range of λ from $1(10^{-6})$ to $6(10^{-6})$ hr^{-1} . Figure 7 shows the 5 best subsystems. For $\lambda < 4(10^{-6})$ hr^{-1} , the $n = 2, k = 1$ subsystem is optimal. For $\lambda > 4(10^{-6})$ hr^{-1} , the $n = 3, k = 1$ subsystem is optimal.

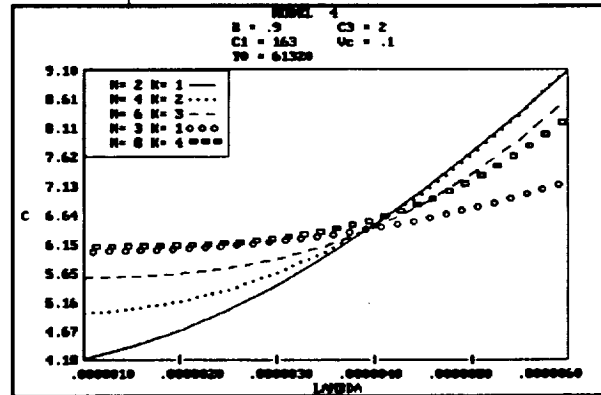


Figure 7

MODEL 5

Suppose we have a situation similar to model 4 but now assume a loss of c_1 if the output fraction from the subsystem is below v_c anytime during the life of the mission.

Model 5 could be applied to recoverable systems, systems which have inherent salvage value or manned systems. Examples include manned aircraft or spacecraft, recoverable undersea vehicles or spacecraft. Model 5 implies that if the output fraction of the subsystem falls below the critical value v_c , something catastrophic will occur, such as loss of the whole system or loss of life. With these systems, loss or significant degradation of a critical subsystem might cause loss of the craft and occupants. An example of such a loss function is given by figure 8.

With this loss function, for $x < kv_c$, $b_0 = c_2$ and $b_1 = 0$ and for $kv_c \leq x \leq k-1$, we have $d(x/k) = 1 - x/k$, $m = 1$, $b_0 = a$ and $b_1 = -aT_0^{-1}$ where $a = c_2(1 - v_c)^{-1}$ with $0 < v_c < 1$.

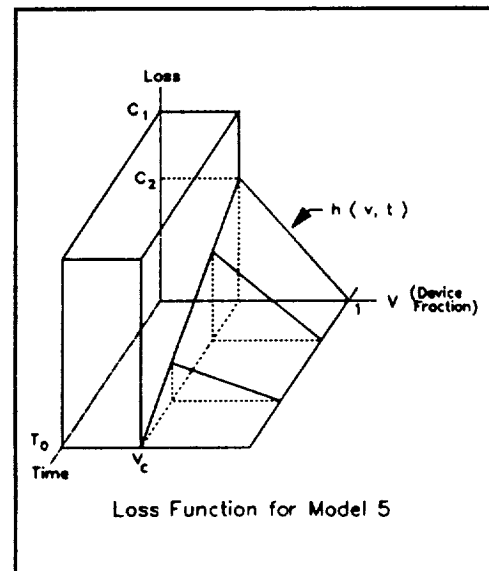


Figure 8

Using (5)

$$C = n c_0 g(k) / (k+r) \left\{ c_1 \sum_{x=0}^{x=kv_0} \binom{n}{x} [\exp(-\lambda T_0)]^x [1 - \exp(-\lambda T_0)]^{n-x} \right. \\ \left. + a \sum_{x=kv_0}^{k-1} J(x)(1-x/k) A \right. \\ \left. - a T_0^{-1} \sum_{x=kv_0}^{k-1} J(x)(1-x/k) \sum_{i=0}^{n-x-1} (-1)^{i+1} \binom{n-x-1}{i} B \right\}$$

Use of the BASIC program is applicable to view C over a range of either λ or c_1 .

BASIC PROGRAMS

The authors will be sending copies of the BASIC program to selected organizations in the United States for initial testing. It is anticipated that the Basic program will become available in the future through NASA's Computer Software Management and Information Center (COSMIC).

SUMMARY

Table 1 contains a summary of the five models which can be applied in a redundancy cost analysis.

Table 1

Redundancy Cost Models Considered in this Paper

Model 1	Simplest cost model. The subsystem consists of n modules, of which k are required for success of the mission. If less than k modules are good, a loss of c_1 occurs. In model 1, k is fixed.
Model 2	Same as model 1 except k may also vary. The $g(k)$ cost function is also available to be used where increased redundancy brings in more (non-linear) cost.
Model 3	Model 3 expands on models 1 and 2. Linear (or other) loss functions are utilized. If less than k modules are good, some loss will occur but not necessarily the entire loss of c_1 . The loss which occurs depends upon some critical output fraction v_c .
Model 4	Model 4 considers time in the loss function. Modules in the subsystem fail exponentially with rate λ .
Model 5	Model 5 handles situations where output fraction below v_c causes a loss which is not time dependent, e.g., manned space missions where loss of a major portion of a critical subsystem may cause loss of life.

REFERENCES

- [1] Suich, R. and Patterson, R., "Balancing Reliability and Cost to Choose the Best Power Subsystem", NASA Technical Memorandum 104453, 1991.
- [2] Suich, R. and Patterson, R., "K-out-of-n: G Systems: Some Cost Considerations", IEEE Transactions on Reliability, Vol. 40, No. 3, 1991.
- [3] Suich, R. and Patterson, R., "How Much Redundancy? - Some Cost Considerations, Including Examples for Spacecraft Systems", NASA Technical Memorandum 103197, 1990.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.</small>				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE		3. REPORT TYPE AND DATES COVERED Technical Memorandum
4. TITLE AND SUBTITLE Reliability and Cost: A Sensitivity Analysis			5. FUNDING NUMBERS WU-506-41-41	
6. AUTHOR(S) Ronald C. Suich and Richard L. Patterson				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191			8. PERFORMING ORGANIZATION REPORT NUMBER E-6631	
9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, D.C. 20546-0001			10. SPONSORING/MONITORING AGENCY REPORT NUMBER NASA TM - 105293	
11. SUPPLEMENTARY NOTES Ronald C. Suich, California State University, Fullerton, California 92634 (work funded by NASA Grant NAG3-1100). Richard L. Patterson, NASA Lewis Research Center. Responsible person, Richard L. Patterson, (216) 433-8166.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified - Unlimited Subject Category 83			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) In the design phase of a system, how does a design engineer or manager choose between a subsystem with .990 reliability and a more costly subsystem with .995 reliability? When is the increased cost justified? High reliability is not necessarily an end in itself but may be desirable in order to reduce the expected cost due to subsystem failure. However, this may not be the wisest use of funds since the expected cost due to subsystem failure is not the only cost involved. The subsystem itself may be very costly. We should not consider either the cost of the subsystem or the expected cost due to subsystem failure separately but should minimize the total of the <u>two costs</u> , i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure.				
14. SUBJECT TERMS Reliability; Redundancy; Space; Cost analysis; System analysis			15. NUMBER OF PAGES 14	
			16. PRICE CODE A03	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	

THE INTERNATIONAL BANK

National Aeronautics and
Space Administration

Lewis Research Center
Cleveland, Ohio 44135

Official Business
Penalty for Private Use \$300

FOURTH CLASS MAIL

ADDRESS CORRECTION REQUESTED



Postage and Fees Paid
National Aeronautics and
Space Administration
NASA 451

NASA
